PHY 610 (Quantum Fields I)

Quantum Fields I

Spring 2026

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Teaching Assistant:

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Syllabus

Requirements

I will assume that the students are familiar with the basics of Special Relativity, Quantum Mechanics, and Statistical Mechanics. Familiarity with Complex Analysis (analytic continuation, contour integration) will also be assumed.

Lecture Notes

I will be posting Lecture Notes during the course, either prior of shortly after each lecture.

Recommended textbooks:

I am not going to follow closely any particular textbook. But the main reading to go with this course is

> **M.Peskin, D.Schroeder, An Introduction To Quantum Field Theory, Addison-Wesley 1995**
>
> (http://www.fulviofrisone.com/attachments/article/483/Peskin, Schroeder -An introduction To Quantum Field Theory(T).pdf)
> The textbook provides complimentary treatment of a number of subjects, and working through some chapters is highly recommended. Parts important for following this course will be indicated during the lectures. I will refer to this book as **PS**.

Some parts of my discussion are borrowed from

> **J-B.Zuber, C.Itzykson, "Quantum Field Theory", Dover Books on Physics, 2012**
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> **J.Zinn-Justin "Quantum Field Theory and Critical Phenomena", A Clarendon Press, 2002**
>
> **K.G. Wilson, John B. Kogut, The Renormalization group and the epsilon expansion, Phys.Rept. 12 (1974) 75-200**

Homeworks and Exercises

Homework Problems

I will be posting about 12 homework problems during the course. I request that solution for each problem be submitted within two weeks from the date it is given. Preferred way to submit your solution is either digital, or scanned handwritten notes. If these ways of submitting are problematic for you, please let me know and we will make special arrangements.

Exercises

Every now and then I will suggest "Exercices". Those are like homework problems, but I will not require you submitting your work. Nonetheless, working the Exercises out is highly recommended.

Final Exam

There will be no mid-term exam. There will be a Final Exam, in the form of a take home task, posted some time close to the end of regular classes. You will be given at least a week to work on the exam. I expect the same mode of turning the exams in as with the Homework Problems.

Grades

I will derive your grade on the basis of your homeworks [%50], final exam work [%40], and in class activity [%10]. I will inform you about the grade couple of days before the deadline for the grade roaster submission, so that you will have some time to complain about the grades.

Course outline

- Classical field theory. Lagrangian density. Field equations. Symmetries and Noether's theorem. Integrals of motion. Energy-momentum tensor. Klein-Gordon field.

- Canonical quantization of fields. Quantum Klein-Gordom model. Space of states and Hamiltonian. Particles. Vacuum energy. Cutoff and counterterm.

- Casimir effect. Calculation of the vacuum energy. Divergent sum. Regularization and counterterm. Math: Euler-McLaurin summation formula.

- Quantum Klein-Gordon field again. Energy spectrum. Particle interpretation. Heisenberg field operators. Commutators. Analyticity at complex values of time. Local commutativity. Time ordering. Klein-Gordon propagator. Imaginary time and Euclidean quantum field theory.

- Path integral in quantum mechanics. Representation of the time evolution operator. Continuation to imaginary time. Imaginary time path integral. Relation to classical statistical mechanics. Time evolution operator as the partition sum.

- Path integral in quantum mechanics again. Correlation functions. Relation to ground state expectation values of the Heisenberg operators. Role of the time ordering. Path integral for relativistic particle. Casuality. Imaginary time and Euclidean space-time.

- Path integral in Klein-Gordon theory. Correlation functions. Wick's theorem. Recursion relation. Interaction. Example: $varphi^4$ theory. Schwinger-Dyson equation.

- Perturbation theory. Perturbative expansion for correlation functions. Wick's contractions and Feynman diagrams. Multiplicity counting and symmetry factors. Feynman rules for φ^4. Connected vs disconnected diagrams. Factorization of the vacuum fragments. Vacuum diagrams. Partition function of Euclidean field theory and vacuum energy in real time.

- Feynman diagrams (Continued). Connected and disconnected diagrams again. Generating functional. One-particle irreducible diagrams. Proper vertices.

- Effective action. Functional Legendre transform. Relation to proper vertices. Semiclassical (loop) expansion. Tree diagrams. Effective action in the tree approximation.

- Loop expansion (Continued). One-loop correction. Gaussian functional integral and functional determinant. Diagram representation. Effective action vs Generating functional revisited.

- Momentum space correlation functions and proper vertices. Feynman rules in the momentum space. Leading contributions to the mass operator. Divergency and regularization. Mass counterterm. Divergences in φ^4. Superficial degree of divergence. Primitive divergences.

- Primitive divergences in φ^4. Counterterms and renormalized parameters. Renormalization program. Renormalized perturbation theory. Divergences in scalar theories. Role of the space-time dimensionality. Perturbatively Super-renormalizable, Renormalizable, and Non-renormalizable theories.

- Systematics of renormalization. Regularization methods. Dimensional regularization. Renormalizations in φ^4, revisited. Renormalization schemes. Normalization conditions.

- Renormalization at the one-loop order. One loop diagrams. Feynman parametrization. Evaluation of the momentum integrals. Counterterms in dimensional regularization. Minimal subtraction scheme. Renormalization of composite fields in φ^4. Two-loop analysis.

- Perturbative renormalization group. Finite renormalizations. Short distance problem. Massless φ^4. Critical surface in the parameter space. Renormalization as the scale transformation. Callan-Symanzik equation. Renormalization group beta function.

- Renormalization group flow. Calculation of β and γ functions in φ^4. Mass perturbation. Wilson's renormalization group. Relation to second order phase transitions. Critical behavior and critical exponents. Landau theory.

- Fluctuational theory of phase transitions. Landau-Ginzburg model. Relation to φ^4. Wilson's renormalization group. Slow vs fast modes. Integrating out the fast modes. Quasi-local actions. Change of scale. Wilson's RG transformation.

- Wilson's RG (Continued). Space of quasi-local actions. RG flow equation. Topological properties of renormalization group flow. Fixed points.

- Critical fixed points. Linearization of the RG flow near a critical fixed point. Critical surface. Unstable manifold. Scale invariance at the fixed point. Anomalous dimensions. Relevant vs irrelevant operators. Marginal operators. Gaussian fixed point. φ^4 flow at $d = 4$.

- RG flow and Poincare-Dulac normal form. Resonances. Callan-Symanzik equation revisited. Fixed point and criticality. Critical exponents.

- Wilson-Fisher fixed point. Generic d. Epsilon expansion. $O(N)$ scalar theory. Spontaneous breakdown of the symmetry. Goldstone bosons.

- Non-linear sigma model at $d = 2 + \epsilon$. Beta function. Asymptotic freedom.

- RG and Energy-Momentum tensor. Callan-Symanzik again. Fixed point theory: Conformal invariance. Scale invariance and density of states.

- Operator formalism vs Functional Integral in QFT. Correlation functions and Energy-Momentum Tensor. Space of states. Reflection positivity.

- Energy-Momentum operators. Stationary states. Energy positivity. Analyticity of the configuration-space correlation functions. Energy-Momentum spectrum. Particle states. K allen-Lehmann representation.

- Asymprotic states. S-matrix. LSZ reduction formula.

- LSZ formula and crossing symmetry.

General Statements

If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact the Student Accessibility Support Center, Stony Brook Union Suite 107, (631) 632-6748, or at sasc@stonybrook.edu. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential.

[In addition, this statement on emergency evacuation is often included, but not required:] Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and the Student Accessibility Support Center. For procedures and information, visit Environmental Health and Safety.

Academic Integrity Statement

Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty is required to report any suspected instances of academic dishonesty to the Academic Judiciary. Faculty in the Health Sciences Center (School of Health Professions, Nursing, Social Welfare, Dental Medicine) and School of Medicine are required to follow their school-specific procedures. For more comprehensive information on academic integrity, including categories of academic dishonesty please refer to the academic judiciary website at
http://www.stonybrook.edu/commcms/academic_integrity/index.html

Critical Incident Management

Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Student Conduct and Community Standards any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn. Faculty in the HSC Schools and the School of Medicine are required to follow their school-specific procedures. Further information about most academic matters can be found in the Undergraduate Bulletin, the Undergraduate Class Schedule, and the Faculty-Employee Handbook.

Accessibility

Stony Brook University is committed to ensuring a supportive, inclusive, and equitable learning environment for all members of our community, which includes upholding the principles of Title II of the Americans with Disabilities Act (ADA). As a student in this course, it is important to recognize your role in ensuring that all classmates, including those who use assistive technologies, can fully engage with and comprehend the course content. Therefore, any digital materials you create and share, such as assignments, presentations, or shared documents, must be designed to be digitally accessible using the most up to date version of the WCAG 2.1 Level AA guidelines. Accessible practices include, but are not limited to, providing alternative text for images, using clear heading structures, and ensuring captions for any video or audio you incorporate. Guidance and assistance in making your digital content accessible can be found on the Accessibility Resources for Students page. For additional support, please visit Deque University.