Computing tone: Input Strict Locality over Multiple Inputs

Contribution: What is the nature of autosegmental computation in phonology? Computational characterizations highlight the trade-offs between data structures and computational expressivity required of phonological grammars. Here we demonstrate a class of algebraic functions and their automata that are both local over multiple-input strings, and necessary for computing the range of tonal processes.

Background: Segmental phonology is normally represented with linear strings of symbols. A majority of segmental phonology is also computationally *local*, both as languages (Strictly Local) and as transformations (Input Strictly Local) (Chandlee, 2014; Chandlee and Heinz, 2018). With this data structure, these local classes of processes can be computed with restricted single-tape finite-state transducers. Under a single linear string representation, tone processes appear significantly more complex than segmental processes (Jardine, 2016). Single strings also fail to capture the suprasegmental nature of tone. For example, Mende has a process of left-to-right tonal spread (Jardine, 2016). Tones and vowels match 1-1 until up until the last tone or vowel: *nìkílì* (1a). If there are more vowels than tones, then the final tone spreads: *félàmà* (1c).[1]

(1) a. nìkílì LHL 'groundnut' b. ngílà HL 'dog' c. félàmà HLL 'junction'

Tone as autosegments: Autosegmental representations (ASR) are a richer data structure that showcase the nonlinear nature of tone by breaking up a linear string into parallel strings or tiers (tone and vowel/mora), often represented with graphs (see Figure (1)). Jardine (2017, 2019) showed that computing well-formedness for tonal structures is Strictly Local over such ASRs as *graphs*, and for transformations, Koser et al. (2019) showed that mapping ASRs without preassociation to ASRs with associations is likewise a local process. Chandlee and Jardine (2019) define a class of logical functions over ASRs called Autosegmental Input-Strictly Local functions (A-ISL), which adequately model many tone mappings that lack preassociation, but cannot generate some tone mappings with preassociation.

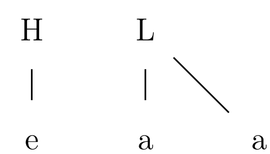

Figure 1: ASR for *félànà*

Multi-Input locality: A-ISL functions are defined logically, and do not have the precise algebraic or automata-theoretic advantages that other Subregular classes enjoy. We define the *Multi-Input Strictly Local* functions. Intuitively, M-ISL functions have as input tuples of strings (n-strings) encoding multiple input items, and produce a single coherent output. We prove an equivalence between M-ISL functions and a class of asynchronous multi-tape transducers, a type of machine used before for tone (Kay, 1987; Wiebe, 1992). The M-ISL class is a strict superclass of both ISL and A-ISL, and as such can handle the range tone patterns they can, and some that they cannot. Additionally, this explicitly connects logically defined subregular functions with algebraic and automata-theoretic characterizations over enriched data structures.

M-ISL tone example: The Mende example can be computed by the M-ISL multi-tape transducer in Figure (2), where the input consists of an n-string of *Tones* and *Vowels* re-

[1] If there are more tones than vowels, then the final vowel is associated with the remaining tones: *nyáhǎ* has a falling contour F made up of HL. We don't discuss these examples here, but they are still M-ISL.

spectively (abstracting away from consonants). Each transition arc can advance (+1) or stay put (0) on either tape. The tone tape is visualized as above the vowel tape. The transducer models a 2-M-ISL function where $k = 2$ because each state encodes the last $k-1 = 1$ suffixes on *each* input tape (separated by a semicolon). A derivation for *félàmà* (1c) is in Table (1). At step 5, upon reading ⋉on the tone-tape, asynchrony allows the read-head to advance on the vowel tape but not on the tone tape, capturing the spreading effect.

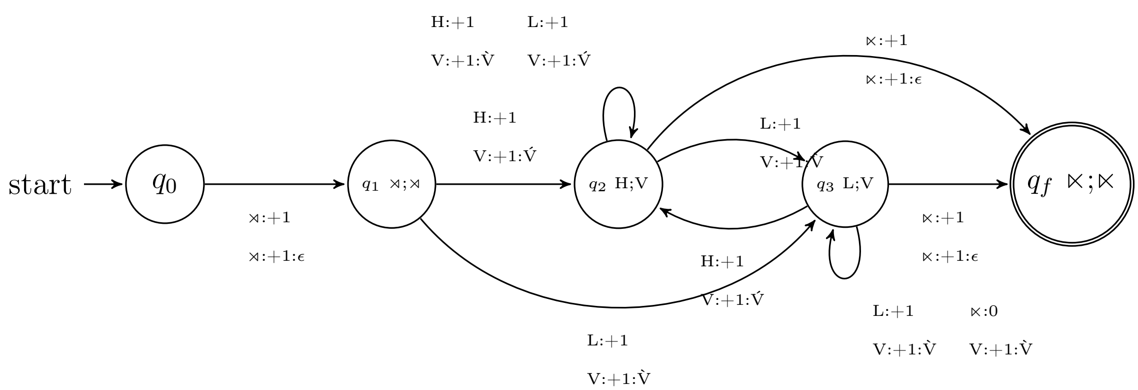

Figure 2: 1-MISL MT-FST for final spread.

	Current state	Tone tape		Vowel tape		Output symbol	Output string
1.	q_0	⋊HL⋉		⋊eaa⋉			
2.	q_1	⋊HL⋉	⋊:+1	⋊eaa⋉	⋊:+1	ϵ	
3.	q_2	⋊HL⋉	H:+1	⋊eaa⋉	e:+1	*é*	*é*
4.	q_3	⋊HL⋉	L:+1	⋊eaa⋉	a:+1	*à*	*éà*
5.	q_3	⋊HL⋉	⋉:0	⋊eaa⋉	a:+1	*à*	*éàà*
6.	q_f	⋊HL⋉	⋉:+1	⋊eaa⋉	⋉:+1	ϵ	*éàà*

Table 1: Derivation table for tone-vowel mapping in *félàmà*

Breadth: Koser et al. (2019) provide a typology of tonal mappings which *lack* preassociation. We show they are also M-ISL. As for functions where the input has preassociated pairs of tones and vowels, Chandlee and Jardine (2019) showed that some but not all are A-ISL. Using an encoding for preassociation adapted from Yli-Jyrä (2013, 2015), we show that various preassociation patterns catalogued in Chandlee and Jardine (2019) are M-ISL. Surprisingly, there are certain functions which are not A-ISL but they are M-ISL. M-ISL is thus strictly a super-class of ISL and A-ISL. Still certain tone functions are neither ISL, A-ISL, or M-ISL, e.g patterns where tones alternate as in /HHHH/→[HLHL].

References: **[1]** Chandlee, J. (2014). Strictly Local Phonological Processes. Ph. D. thesis, University of Delaware, Newark, DE. **[2]** Chandlee, J. and J. Heinz (2018). Strict locality and phonological maps. Linguistic Inquiry 49(1), 23–60. **[3]** Chandlee, J. and A. Jardine (2019). Autosegmental input strictly local functions. Transactions of the Association for Computational Linguistics 7, 157–168. **[4]** Jardine, A. (2016). Computationally, tone is different. Phonology 33(2), 247–283. **[5]** Jardine, A. (2017). The local nature of tone-association patterns. Phonology 34(2), 363–384. **[6]** Jardine, A. (2019). The expressivity of autosegmental grammars. Journal of Logic, Language and Information 28(1), 9–54. **[7]** Kay, M. (1987). Nonconcatenative finite-state morphology. In Third Conference of the European Chapter of the Association for Computational Linguistics. **[8]** Koser, N., C. Oakden, and A. Jardine (2019). Tone association and output locality in non-linear structures. In Supplemental proceedings of AMP 2019. **[9]** Wiebe, B. (1992). Modelling autosegmental phonology with multi-tape finite state transducers. Master's thesis, Simon Fraser University. **[10]** Yli-Jyrä, A. (2013). On finite-state tonology with autosegmental representations. In Proceedings of the 11th international conference on finite state methods and natural language processing. Association for Computational Linguistics. **[11]** Yli-Jyrä, A. (2015). Three equivalent codes for autosegmental representations. In Proceedings of the 12th International Conference on Finite-State Methods and Natural Language Processing 2015 (FSMNLP 2015 Düsseldorf).