Representation and the Computation of Tone Processes

This paper shows that enhancing the representation provides a tight characterization of the computationally complex tone processes noted in Jardine (2016). Jardine argues that some tone processes are more computationally complex than segmental processes by giving a number of examples of *unbounded circumambient* (UC) processes, in which triggers or blockers can be arbitrarily far away on either sides of any target. Unbounded Tone Plateauing (UTP) in Luganda (Hyman and Katamba, 2010) is one example of a UC process. In UTP, H(igh) tones on either side of an unbounded span of toneless TBUs form a single H-toned plateau, as in (1).

(1) tw - áa - láb - **w** - **a wal** ú simbi → tw - áá - láb - **w** - **á wál** ú simbi ‘We saw him, Walusimbi’

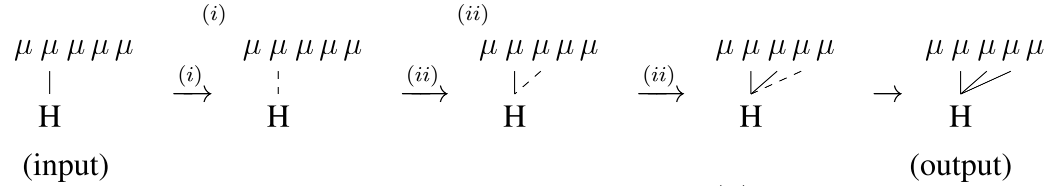
H H H H

Jardine argues that UC processes are not *subsequential*, a class of functions that can be computed over deterministic finite-state transducers (Mohri 1997) that has been argued to form a tight bound on segmental phonology (Heinz and Lai 2013, Heinz 2018). We show that extending a logical notion of subsequentiality to autosegmental representations (ARs; Goldsmith 1976) allows us to capture UC processes in tone, thus providing a sufficiently expressive, yet restrictive characterization of tone. Thus, fixing the logical power while enhancing the representation captures complex processes in a restrictive way (see also Jardine and Heinz 2016).

Logical transductions allow a representation-independent notion of complexity. We use *quantifier-free* logical transductions, which identify *local* structures (Chandlee and Lindell, to appear), and the *least fixed point* operators (Libkin 2004), which identify local structures in the *output*. Over strings, the resulting *quantifier-free least fixed point* (QFLFP) is a subset of the subsequential class (Chandlee and Jardine 2019). Koser et al. (2019) extend QFLFP to ARs. An example is given in (2) for unbounded spread in Cilungu (Bickmore 1996), in which a H tone spreads until the penult.

The formula in (2) uses a logic in which x and y as variables ranging over positions on the timing and melody tiers, respectively; $last(x)$ indicates that x is the last element on its tier, $p(x)$ indicates the immediate predecessor of x, and $\alpha(x)$ indicates the element to which x is associated in the input, and $\alpha'(x)$ indicates the element to which x is associated in the output. Thus, (2) defines $\alpha'(x) \approx y$—that is, when x is associated to y in the output—by listing the conditions that must be true for x and y.

(2) a. $\alpha'(x) \approx y \stackrel{\text{def}}{=} \underbrace{\alpha(x) \approx y}_{(i)} \vee (\underbrace{\alpha'(p(x)) \approx y \wedge \neg last(x)}_{(ii)})$

b. 

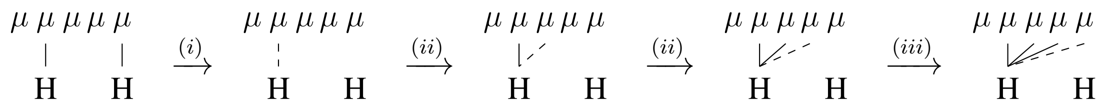
The definition in (2a) reads as follows: the first conjunct $\alpha(x) \approx y$ in (i) is true when x is associated to y (α points x to y) in the *input*, and the second conjunct $\alpha'(p(x)) \approx y \wedge \neg last(x))$ in (ii) is true when the *predecessor* of x ($p(x)$) is associated to y in the *output*, as long as x is non-final ($\neg last(x)$). Thus, the definition states that $a'(x) \approx y$—that is, x is associated to y in the output—if and only if x and y satisfy either disjunct (i) or (ii). A derivation is given in (2b) to show how successive pairs are associated, converging to an output in which H spreads up to the penult.

The Luganda UTP can thus be derived using QFLFP over ARs, as shown below in (3).

(3) a. $R(x,y) \stackrel{\text{def}}{=} (\underbrace{\alpha(x) \approx y \wedge first(y)}_{(i)}) \vee (\underbrace{R(p(x),y) \wedge \neg(last(\alpha(x)))}_{(ii)})$

b. $\alpha'(x) \approx y \stackrel{\text{def}}{=} R(x,y) \vee (\underbrace{last(\alpha(x)) \wedge first(y)}_{(iii)})$

The formula in (3) defines the UTP function by first defining a predicate $R(x,y)$ recursively in (3a) and using that predicate to define the output association relation in (3b). Each labeled disjunct works as follows: (i) states that x and y are associated if x is associated to the first y in the input; or if the predecessor of x on the timing tier is associated to y and it is not the case that the position to which x is associated is last (ii). Finally, (iii) associates the TBU associated to the final tone ($last(\alpha(x))$ to the first tone ($first(y)$). (4) shows a step-by-step derivation of (3). (We omit the deletion of the second H, but this is also QFLFP-definable.)

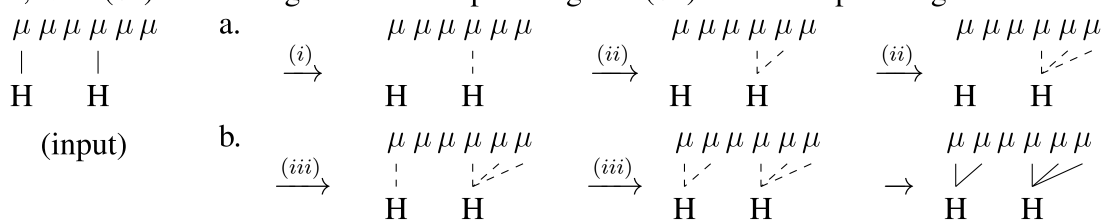
(4)


Another UC tone process is found in Copperbelt Bemba, where the H tone spreads unboundedly to the right unless there is another underlying H that can be arbitrarily far to the right (Bickmore and Kula, 2015). In this case, the first H only spreads up to three TBUs at most, obeying the OCP. The process is illustrated in (6), and the definition is given below in (5).

(5) a. $R(x,y) \overset{\text{def}}{=} (\underbrace{\alpha(x) \approx y \wedge last(y)}_{(i)}) \vee (\underbrace{R(p(x),y)}_{(ii)})$

b. $\alpha'(x) \approx y \overset{\text{def}}{=} R(x,y) \vee ((\underbrace{\alpha(x) \approx y \vee \alpha(p(x)) \approx y \vee \alpha(p(p(x))) \approx y}_{(iii)}) \wedge \underbrace{\neg s(x) \approx s(y)}_{(iv)})$

(5a) describes the unbounded case. Disjunct (i) first considers the pair that are associated in the input *only if* y is the last tone. The disjunct in (ii) then recursively associates subsequent TBUs, analogous to unbounded spreading in (2). The fact that this recursion only begins in (i) with the last tone captures the generalization in Bemba that only the last tone spreads unboundedly. The definition in (5b) then captures the bounded case: (iii) states that x is associated to y if they are associated in the input, *or* if $p(x)$ is associated to y in the input (binary spreading), *or* $p(p(x))$ is associated to y in the input (ternary spreading). Additionally, (iii) is subject to the conjunction in (iv): x and y can only associate when there is no following input association (here, $s(x)$ indicates the immediate *successor* of x). In other words, (iv) is the OCP condition on bounded spreading. (6) gives an example derivation, with (6a) illustrating unbounded spreading and (6b) bounded spreading.

(6)


In conclusion, enriching the representation by shifting from a string-based representation to ARs allows us to use the same logical power for both segmental and tonal processes. Although this work shows that QFLFP over ARs is strictly more expressive than the subsequential class, how much more expressive it is is an open question. While QFLFP logic is restrictive in that recursion can only be defined locally, future work will characterize exactly the range of processes definable by QFLFP over ARs. A related question is whether this class can distinguish Copperbelt Bemba from 'true' sour-grapes spreading (Pater 2018, O'Hara and Smith 2019). Regardless, it is clear that computational study confirms the notion from phonological theory that representation matters.

Selected references: • Heinz, J. (2018). The computational nature of phonological generalizations. In Hyman, L. and Plank, F., editors, *Phonological Typology*. • Koser, N., Oakden, C., and Jardine, A. (2019). Tone association and output locality in non-linear structures. In *Supplemental Proceedings AMP 2019*.