Are Phonological Functions Total or Partial?

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It has been a standard assumption in computational phonology that phonological processes are *total* functions that apply to any possible string built from the segmental inventory (i.e., their domain is Σ^*) (Johnson 1972, Kaplan & Kay 1994, Heinz 2011, Chandlee 2014, Bale & Reiss 2018). The alternative would be that these functions are *partial*, or apply to a restricted domain (i.e., $D \subsetneq \Sigma^*$).

From a theory-internal perspective this distinction between total and partial functions relates to whether or not there are assumed restrictions on underlying forms (or inputs). In SPE (Chomsky & Halle 1968) the use of Morpheme Structure Constraints implies the functions are partial, while in classic Optimality Theory (Prince & Smolensky 1993) the Richness of the Base means the grammar is a total function. Another way this distinction can be drawn is whether or not the function includes *vacuous rule application* or identity maps ($f(x) = x$): omitting such maps amounts to a domain that includes only those strings that satisfy the rule's structural description.

In addition to aligning with the Richness of the Base principle, a motivation for assuming total functions has been that they simplify the operation of composing a set of processes into a single function that maps directly from UR to SR. In addition, in work that aims to classify individual processes (or process types) in terms of their computational complexity, total functions serve to *isolate* the process in question such that assessing whether the input is valid in some way does not factor into the complexity analysis.

In this talk I argue that in some cases a partial function analysis is both possible and advantageous. The advantages include 1) a better characterization of computational complexity and 2) learnability with more plausible training data. These cases currently include non-local phenomena for which the number of intervening segments is bounded by syllabic structure. In particular, I will use vowel dissimilation and vowel harmony to illustrate these advantages of partial functions.

These two types of phenomena are classified as both total and partial functions using the right subsequential (RS) functions and the right output strictly local (ROSL) functions, which form part of the subregular hierarchy of functions. The RS functions are a proper superset of the ROSL functions, and so processes classified as properly RS are more complex than those classified as ROSL. These analyses show that as total functions vowel dissimilation and vowel harmony are subsequential[1], but as partial functions they are ROSL (except in the case of transparent vowels). I argue that the partial function analysis provides a better characterization of these ROSL phenomena, capturing the fact that they are output-oriented and bounded.

[1] Assuming a one-sided context. Circumambient harmony, with a two-sided context, is not subsequential (Jardine 2016, McCollum et al. (under review)).

In addition, I demonstrate how a partial function analysis provides a learnability advantage, in that the OSL function learner (OSLFIA (Chandlee et al. 2015)) successfully identifies the partial function given less and more plausible training data compared to what it needs to learn it as a total function (see also Rasin & Katzir (under review) for an advantage of constraints on URs in the context of MDL learning). The proof of correctness of the OSLFIA assumes the target is a total function, which means it is only guaranteed to learn total functions, not that it can't learn partial ones. Though their formal properties remain to be defined, the types of phonological maps for which a partial function analysis has meaningful consequences may correspond to those that fall within reach of the OSLFIA.

The computational property of strict locality has largely been reserved for processes describable with rules like (1), in which the structural description *cad* is a contiguous substring of bounded length. In rules like (2) and (3), on the other hand, the structural description contains an unbounded amount of material between the trigger and target, either of a particular type (X_0) or a combination of types (...). Such rules have been viewed as computationally non-local and classified as subsequential (Heinz & Lai 2013, Luo 2017, Payne 2017).

(1) a → b / c__d   (2) a → b / __ X_0 d   (3) a → b / __ ... d

More recently, function classes that operate over non-linear representations (Chandlee & Jardine 2019) or a projected tier (Burness & McMullin 2019) have been proposed for non-local processes. Though still subsets of subsequential, these classes have greater complexity and expressivity compared to OSL. The use of partial functions for rules like (2) offers a way to preserve locality and the resultant learning results without adding the complexity of a non-linear representation.

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